Tests for Multivariate Linear Models with the **car** Package

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useR! 2011

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- The `Anova` function in the **car** package (Fox and Weisberg, 2011) can perform partial ("type II" or "type III") tests for the terms in a multivariate linear model, including simply specified multivariate and univariate tests for repeated-measures models.
- The `linearHypothesis` function in the **car** package can test arbitrary linear hypothesis for multivariate linear models, including models for repeated measures.
- Both the `Anova` and `linearHypothesis` functions return a variety of information useful in further computation on multivariate linear models.

A Simple Example: The Anderson-Fisher Iris Data

- Anderson's data on three species of irises in Quebec's Gaspé Peninsula (Anderson, 1935) are a staple of the literature on multivariate statistics, and were used by R. A. Fisher (1936) to introduce discriminant analysis:

```
> library(car)
> some(iris)
```

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
25	4.8	3.4	1.9	0.2	setosa
47	5.1	3.8	1.6	0.2	setosa
67	5.6	3.0	4.5	1.5	versicolor
73	6.3	2.5	4.9	1.5	versicolor
104	6.3	2.9	5.6	1.8	virginica
109	6.7	2.5	5.8	1.8	virginica
113	6.8	3.0	5.5	2.1	virginica
131	7.4	2.8	6.1	1.9	virginica
140	6.9	3.1	5.4	2.1	virginica
149	6.2	3.4	5.4	2.3	virginica

A Simple Example: The Anderson-Fisher Iris Data

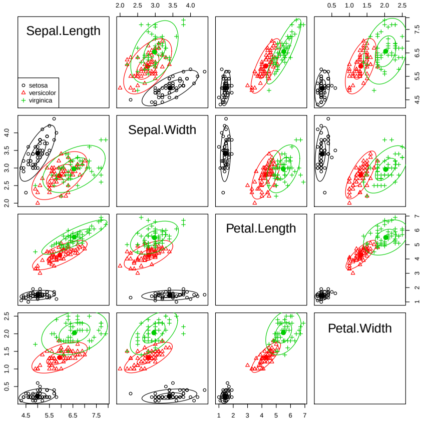
- Three species of irises in the Anderson/Fisher data set: setosa (left), versicolor (center), and Virginica (right)



Source: The Wikimedia Commons.

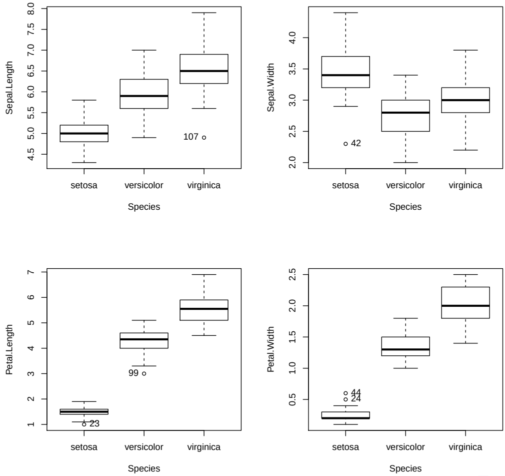
A Simple Example: The Anderson-Fisher Iris Data

```
> scatterplotMatrix(~ Sepal.Length + Sepal.Width + Petal.Length
+      + Petal.Width | Species,
+   data=iris, smooth=FALSE, reg.line=FALSE, ellipse=TRUE,
+   by.groups=TRUE, diagonal="none")
```



A Simple Example: The Anderson-Fisher Iris Data

```
> par(mfrow=c(2, 2))
> for (response in c("Sepal.Length", "Sepal.Width", "Petal.Length",
        "Petal.Width"))
+   Boxplot(iris[, response] ~ Species, data=iris, ylab=response)
```



A Simple Example: The Anderson-Fisher Iris Data

- Fitting a one-way MANOVA model to the iris data:

```
> mod.iris <- lm(cbind(Sepal.Length, Sepal.Width, Petal.Length,
+    Petal.Width) ~ Species, data=iris)

> class(mod.iris)

[1] "mlm" "lm"

> mod.iris

Call:
lm(formula = cbind(Sepal.Length, Sepal.Width, Petal.Length,
    Petal.Width) ~ Species, data = iris)

Coefficients:
                   Sepal.Length  Sepal.Width  Petal.Length  Petal.Width
(Intercept)         5.006         3.428        1.462         0.246
Speciesversicolor   0.930        -0.658        2.798         1.080
Speciesvirginica    1.582        -0.454        4.090         1.780
```

A Simple Example: The Anderson-Fisher Iris Data

- For this simple model, with just one term, `Anova` in **car** and `anova` produce the same MANOVA test:

```
> (manova.iris <- Anova(mod.iris))

Type II MANOVA Tests: Pillai test statistic
        Df test stat approx F num Df den Df Pr(>F)
Species  2      1.19     53.5      8    290 <2e-16

> anova(mod.iris)

Analysis of Variance Table

             Df Pillai approx F num Df den Df Pr(>F)
(Intercept)   1  0.993     5204      4    144 <2e-16
Species       2  1.192       53      8    290 <2e-16
Residuals   147
```

A Simple Example: The Anderson-Fisher Iris Data

- The `summary` method for `Anova.mlm` objects provides more detail:

```
> summary(manova.iris)

Type II MANOVA Tests:

Sum of squares and products for error:
             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length       38.956      13.630       24.625       5.645
Sepal.Width        13.630      16.962        8.121       4.808
Petal.Length       24.625       8.121       27.223       6.272
Petal.Width         5.645       4.808        6.272       6.157
```

(output continued . . .)

(. . . output concluded)

```
Term: Species

Sum of squares and products for the hypothesis:
             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length        63.21      -19.95       165.25       71.28
Sepal.Width        -19.95       11.34       -57.24      -22.93
Petal.Length       165.25      -57.24       437.10      186.77
Petal.Width         71.28      -22.93       186.77       80.41

Multivariate Tests: Species
                 Df test stat approx F num Df den Df Pr(>F)
Pillai            2      1.19     53.5      8    290 <2e-16
Wilks             2      0.02    199.1      8    288 <2e-16
Hotelling-Lawley  2     32.48    580.5      8    286 <2e-16
Roy               2     32.19   1167.0      4    145 <2e-16
```

A Simple Example: The Anderson-Fisher Iris Data

- The photographs, scatterplot matrix, and boxplots suggest that versicolor and virginica are more similar to each other than either is to setosa.
- The `linearHypothesis` function in **car** can be used to test more specific linear hypotheses about the parameters of a MLM.
- For example, to test for differences between setosa (the baseline level of `Species` and the average of versicolor and virginica:

```
> linearHypothesis(mod.iris,
+     "0.5*Speciesversicolor + 0.5*Speciesvirginica = 0")
```

A Simple Example: The Anderson-Fisher Iris Data

```
Sum of squares and products for the hypothesis:
             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length     52.58453   -23.27787     144.1888    59.86933
Sepal.Width     -23.27787    10.30453     -63.8288   -26.50267
Petal.Length    144.18880   -63.82880     395.3712   164.16400
Petal.Width      59.86933   -26.50267     164.1640    68.16333

Sum of squares and products for error:
             Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length      38.9562     13.6300      24.6246      5.6450
Sepal.Width       13.6300     16.9620       8.1208      4.8084
Petal.Length      24.6246      8.1208      27.2226      6.2718
Petal.Width        5.6450      4.8084       6.2718      6.1566

Multivariate Tests:
                 Df test stat approx F num Df den Df     Pr(>F)
Pillai            1  0.967269 1063.871      4    144 < 2.22e-16 ***
Wilks             1  0.032731 1063.871      4    144 < 2.22e-16 ***
Hotelling-Lawley  1 29.551969 1063.871      4    144 < 2.22e-16 ***
Roy               1 29.551969 1063.871      4    144 < 2.22e-16 ***
```

A Simple Example: The Anderson-Fisher Iris Data

- An equivalent more direct approach is to fit the model with custom contrasts, and then to test each contrast:

```
> C <- matrix(c(1, -0.5, -0.5,  0, 1, -1), 3, 2)
> colnames(C) <- c("set v. vers & virg",
+                  "vers v. virg")
> contrasts(iris$Species) <- C
> contrasts(iris$Species)

           set v. vers & virg vers v. virg
setosa                    1.0            0
versicolor               -0.5            1
virginica                -0.5           -1

> mod.iris.2 <- update(mod.iris)
> rownames(coef(mod.iris.2))

[1] "(Intercept)"                "Speciesset v. vers & virg"
[3] "Speciesvers v. virg"
```

A Simple Example: The Anderson-Fisher Iris Data

```
> linearHypothesis(mod.iris.2, c(0, 1, 0)) # set v. vers & virg

. . .

Multivariate Tests:
                 Df test stat approx F num Df den Df     Pr(>F)
Pillai            1  0.967269 1063.871      4    144 < 2.22e-16 ***
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Roy               1 29.551969 1063.871      4    144 < 2.22e-16 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

Handling Repeated Measures

- *Repeated-measures data* arise when multivariate responses represent the same individuals measured on a response variable (or variables) on different occasions or under different circumstances.

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- There may be a more or less complex design on the repeated measures.
- The simplest case is that of a single repeated-measures or *within-subjects* factor.

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- Repeated-measures designs can be handled with the `anova` function, but it is simpler to get common tests from the `Anova` and `linearHypothesis` functions in the **car** package.

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  - The general procedure is first to fit a multivariate linear models with all of the repeated measures as responses.
  - Then an artificial data frame is created in which each of the repeated measures is a row and in which the columns represent the repeated-measures factor or factors.

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- Repeated-measures designs can be handled with the `anova` function, but it is simpler to get common tests from the `Anova` and `linearHypothesis` functions in the **car** package.
  - The general procedure is first to fit a multivariate linear models with all of the repeated measures as responses.
  - Then an artificial data frame is created in which each of the repeated measures is a row and in which the columns represent the repeated-measures factor or factors.
  - Finally, the `Anova` or `linearHypothesis` function is called, using the `idata` and `idesign` arguments (and optionally the `icontrasts` argument)—or alternatively the `imatrix` argument to `Anova` or `P` argument to `linearHypothesis`—to specify the intra-subject design.

Handling Repeated Measures

- To illustrate, I employ contrived data reported by O'Brien and Kaiser (1985) in "an extensive primer" for the MANOVA approach to repeated-measures designs.
- The data set `OBrienKaiser` is provided by the **car** package:

```
> some(OBrienKaiser, 6)
   treatment gender pre.1 pre.2 pre.3 pre.4 pre.5 post.1 post.2 post.3
2    control      M     4     4     5     3     4      2      2      3
4    control      F     5     4     7     5     4      2      2      3
6          A      M     7     8     7     9     9      9      9     10
7          A      M     5     5     6     4     5      7      7      8
11         B      M     3     3     4     2     3      5      4      7
12         B      M     6     7     8     6     3      9     10     11
   post.4 post.5 fup.1 fup.2 fup.3 fup.4 fup.5
2       5      3     4     5     6     4     1
4       5      3     4     4     5     3     4
6       8      9     9    10    11     9     6
7      10      8     8     9    11     9     8
11      5      4     5     6     8     6     5
12      9      6     8     7    10     8     7
```

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  - *phase*, with three levels for the *pretest*, *post-test*, and *follow-up* phases of the study.

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  - `treatment`, with levels `A`, `B`, and `control`. I will imagine that the treatments `A` and `B` represent different innovative methods of teaching reading to learning-disabled students, and that the `control` treatment represents a standard method.
- The 15 response variables in the data set represent two crossed within-subjects factors:
  - *phase*, with three levels for the *pretest*, *post-test*, and *follow-up* phases of the study.
  - *hour*, representing five successive hours, at which measurements of reading-comprehension are taken within each phase.

Handling Repeated Measures

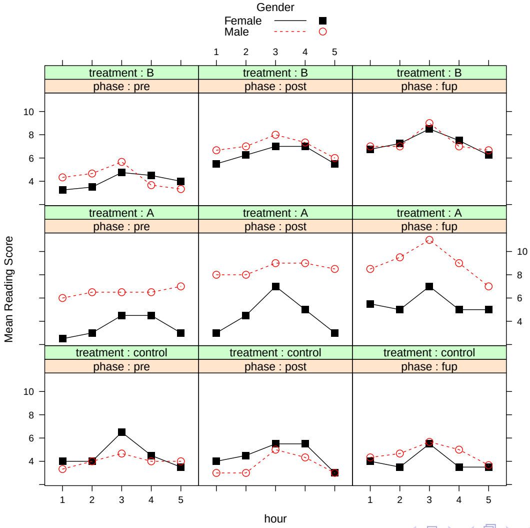
- The data are "unbalanced," with unequal numbers of subjects in the cells of the between-subject design:

```
> xtabs(~ treatment + gender, data=OBrienKaiser)

         gender
treatment F M
  control 2 3
  A       2 2
  B       4 3
```

Handling Repeated Measures

- Mean reading scores for combinations of `gender`, `treatment`, `phase`, and `hour`:



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- There is a possibly quadratic relationship of reading to hour within each phase, with an initial rise and then decline, perhaps representing fatigue, suggesting an hour main effect.
- Males and females respond similarly to the control and B treatment groups, but that males do better than females in the A treatment group, suggesting a possible gender-by-treatment interaction.

- Both of the between-subjects factors have predefined contrasts, with $-1, 1$ "deviation" coding for `gender` (produced by `contr.sum`) and custom contrasts for `treatment`.
- For `treatment`, the first contrast is for the `control` group vs. the average of the experimental groups, and the second contrast is for treatment `A` vs. treatment `B`.

```
> contrasts(OBrienKaiser$treatment)

        [,1] [,2]
control   -2    0
A          1   -1
B          1    1

> contrasts(OBrienKaiser$gender)

  [,1]
F    1
M   -1
```

Handling Repeated Measures

- I define the "data" for the within-subjects design as follows:

```
> phase <- factor(rep(c("pretest", "posttest", "followup"), each=5),
+     levels=c("pretest", "posttest", "followup"))
> hour <- ordered(rep(1:5, 3))
> idata <- data.frame(phase, hour)
> idata
      phase hour
1   pretest    1
2   pretest    2
. . .
5   pretest    5
6  posttest    1
7  posttest    2
. . .
10 posttest    5
11 followup    1
12 followup    2
. . .
15 followup    5
```

Handling Repeated Measures

- Fitting the MLM and calling `Anova` for the repeated-measures MANOVA:

```
> mod.ok <- lm(cbind(pre.1, pre.2, pre.3, pre.4, pre.5,
+                    post.1, post.2, post.3, post.4, post.5,
+                    fup.1, fup.2, fup.3, fup.4, fup.5) ~  treatment*gender,
+                 data=OBrienKaiser)

(av.ok <- Anova(mod.ok, idata=idata, idesign=~phase*hour, type=3))
```

Handling Repeated Measures

```
Type III Repeated Measures MANOVA Tests: Pillai test statistic
                              Df test stat approx F num Df den Df   Pr(>F)
(Intercept)                    1     0.967    296.4      1     10 9.2e-09
treatment                      2     0.441      3.9      2     10 0.05471
gender                         1     0.268      3.7      1     10 0.08480
treatment:gender               2     0.364      2.9      2     10 0.10447
phase                          1     0.814     19.6      2      9 0.00052
treatment:phase                2     0.696      2.7      4     20 0.06211
gender:phase                   1     0.066      0.3      2      9 0.73497
treatment:gender:phase         2     0.311      0.9      4     20 0.47215
hour                           1     0.933     24.3      4      7 0.00033
treatment:hour                 2     0.316      0.4      8     16 0.91833
gender:hour                    1     0.339      0.9      4      7 0.51298
treatment:gender:hour          2     0.570      0.8      8     16 0.61319
phase:hour                     1     0.560      0.5      8      3 0.82027
treatment:phase:hour           2     0.662      0.2     16      8 0.99155
gender:phase:hour              1     0.712      0.9      8      3 0.58949
treatment:gender:phase:hour    2     0.793      0.3     16      8 0.97237
```

Handling Repeated Measures

- Following O'Brien and Kaiser, I report type-III tests, which are computed correctly because the contrasts employed for `treatment` and `gender`, and hence their interaction, are orthogonal in the row-basis of the between-subjects design.

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- When the `idata` and `idesign` arguments are specified, `Anova` automatically constructs orthogonal contrasts for different terms in the within-subjects design, using `contr.sum` for a factor such as `phase` and `contr.poly` for an ordered factor such as `hour`.

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- Following O'Brien and Kaiser, I report type-III tests, which are computed correctly because the contrasts employed for `treatment` and `gender`, and hence their interaction, are orthogonal in the row-basis of the between-subjects design.
- When the `idata` and `idesign` arguments are specified, `Anova` automatically constructs orthogonal contrasts for different terms in the within-subjects design, using `contr.sum` for a factor such as `phase` and `contr.poly` for an ordered factor such as `hour`.
- Alternatively, the user can assign contrasts to the columns of the intra-subject data, either directly or via the `icontrasts` argument to `Anova`. `Anova` checks that the within-subjects contrast coding for different terms is orthogonal.

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- The results show that the anticipated `hour` effect is statistically significant.

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- The `treatment` × `phase` and `treatment` × `gender` interactions are not quite significant.
- There is, however, a statistically significant `phase` main effect.

Handling Repeated Measures

- The results show that the anticipated `hour` effect is statistically significant.
- The `treatment` $\times$ `phase` and `treatment` $\times$ `gender` interactions are not quite significant.
- There is, however, a statistically significant `phase` main effect.
- We should not over-interpret these results, partly because the data set is small and partly because it is contrived.

Handling Repeated Measures

- The `summary` method for `Anova.mlm` objects can report a variety of information, including a traditional "univariate" repeated-measures ANOVA with tests of sphericity and corrections for non-sphericity.

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- The `summary` method for `Anova.mlm` objects can report a variety of information, including a traditional "univariate" repeated-measures ANOVA with tests of sphericity and corrections for non-sphericity.
- Suppressing the multivariate tests:

Handling Repeated Measures

```
> summary(av.ok, multivariate=FALSE)
Univariate Type III Repeated-Measures ANOVA Assuming Sphericity

                              SS num Df Error SS den Df      F  Pr(>F)
(Intercept)                 6759      1    228.1     10 296.39 9.2e-09
treatment                    180      2    228.1     10   3.94  0.0547
gender                        83      1    228.1     10   3.66  0.0848
treatment:gender             130      2    228.1     10   2.86  0.1045
phase                        130      2     80.3     20  16.13 6.7e-05
treatment:phase               78      4     80.3     20   4.85  0.0067
gender:phase                   2      2     80.3     20   0.28  0.7566
treatment:gender:phase        10      4     80.3     20   0.64  0.6424
hour                         104      4     62.5     40  16.69 4.0e-08
treatment:hour                 1      8     62.5     40   0.09  0.9992
gender:hour                    3      4     62.5     40   0.45  0.7716
treatment:gender:hour          8      8     62.5     40   0.62  0.7555
phase:hour                    11      8     96.2     80   1.18  0.3216
treatment:phase:hour           7     16     96.2     80   0.35  0.9901
gender:phase:hour              9      8     96.2     80   0.93  0.4956
treatment:gender:phase:hour   14     16     96.2     80   0.74  0.7496
```

(. . . output continued)

```
Mauchly Tests for Sphericity

                               Test statistic p-value
phase                                   0.749   0.273
treatment:phase                         0.749   0.273
gender:phase                            0.749   0.273
treatment:gender:phase                  0.749   0.273
hour                                    0.066   0.008
treatment:hour                          0.066   0.008
gender:hour                             0.066   0.008
treatment:gender:hour                   0.066   0.008
phase:hour                              0.005   0.449
treatment:phase:hour                    0.005   0.449
gender:phase:hour                       0.005   0.449
treatment:gender:phase:hour             0.005   0.449
```

(. . . output continued)

```
Greenhouse-Geisser and Huynh-Feldt Corrections
 for Departure from Sphericity

                            GG eps Pr(>F[GG])
phase                         0.80    0.00028
treatment:phase               0.80    0.01269
gender:phase                  0.80    0.70896
treatment:gender:phase        0.80    0.61162
hour                          0.46   0.000098
treatment:hour                0.46    0.97862
gender:hour                   0.46    0.62843
treatment:gender:hour         0.46    0.64136
phase:hour                    0.45    0.33452
treatment:phase:hour          0.45    0.93037
gender:phase:hour             0.45    0.44908
treatment:gender:phase:hour   0.45    0.64634
```

Handling Repeated Measures

(. . . output concluded)

```
                           HF eps Pr(>F[HF])
phase                       0.928    0.00011
treatment:phase             0.928    0.00844
gender:phase                0.928    0.74086
treatment:gender:phase      0.928    0.63200
hour                        0.559   0.000023
treatment:hour              0.559    0.98866
gender:hour                 0.559    0.66455
treatment:gender:hour       0.559    0.66930
phase:hour                  0.733    0.32966
treatment:phase:hour        0.733    0.97523
gender:phase:hour           0.733    0.47803
treatment:gender:phase:hour 0.733    0.70801
```

Handling Repeated Measures

- As for simpler multivariate linear models, the `linearHypothesis` function can be used to test more focused hypotheses about the parameters of repeated-measures models, including for within-subjects terms.
- For example, to duplicate the test for the `hour` main effect, we can proceed as follows, testing the intercept in the between-subjects model and specifying the `idata`, `idesign`, and `iterms` arguments to `linearHypothesis`:

```
> linearHypothesis(mod.ok, "(Intercept) = 0", idata=idata,
+   idesign=~phase*hour, iterms="hour") # test hour main effect
. . .
Multivariate Tests:
                 Df test stat approx F num Df den Df    Pr(>F)
Pillai            1     0.933    24.32      4      7 0.000334
Wilks             1     0.067    24.32      4      7 0.000334
Hotelling-Lawley  1    13.894    24.32      4      7 0.000334
Roy               1    13.894    24.32      4      7 0.000334
```

- Alternatively and equivalently, we can generate the response-transformation matrix P for the hypothesis directly:

```
> (Hour <- model.matrix(~ hour, data=idata))

   (Intercept)     hour.L  hour.Q     hour.C  hour^4
1            1 -6.325e-01  0.5345 -3.162e-01  0.1195
2            1 -3.162e-01 -0.2673  6.325e-01 -0.4781
3            1 -3.288e-17 -0.5345  2.165e-16  0.7171
. . .
14           1  3.162e-01 -0.2673 -6.325e-01 -0.4781
15           1  6.325e-01  0.5345  3.162e-01  0.1195

> linearHypothesis(mod.ok, "(Intercept) = 0",
+   P=Hour[ , c(2:5)]) # test hour main effect (equivalent)
```

(output omitted)

- These tests simply duplicate part of the output from `Anova`, but suppose that we want to test the individual polynomial components of the `hour` main effect, such as the quadratic component:

```
> linearHypothesis(mod.ok, "(Intercept) = 0",
+       P=Hour[ , 3, drop=FALSE]) # quadratic
```

```
 Response transformation matrix:
       hour.Q
pre.1   0.5345
pre.2  -0.2673
pre.3  -0.5345
...
fup.4  -0.2673
fup.5   0.5345
```

(output continued . . .)

Handling Repeated Measures

(. . . output concluded)

```
Sum of squares and products for the hypothesis:
       hour.Q
hour.Q  234.1

Sum of squares and products for error:
       hour.Q
hour.Q  46.64

Multivariate Tests:
                 Df test stat approx F num Df den Df    Pr(>F)
Pillai            1     0.834    50.19      1     10 0.0000336
Wilks             1     0.166    50.19      1     10 0.0000336
Hotelling-Lawley  1     5.019    50.19      1     10 0.0000336
Roy               1     5.019    50.19      1     10 0.0000336
```

References

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