Estimating a multivariate normal covariance matrix subject to a Loewner ordering

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Optimization of functions with matrix arguments appear in many statistical contexts. We consider the problem of determining the maximum likelihood estimate of the covariance matrix $\mathbf{\Sigma}$ of a multivariate normal distribution subject to simultaneous Loewner order constraints; for two positive semidefinite $\mathbf{L}$ and $\mathbf{U}$ matrices, we require that $\mathbf{\Sigma} - \mathbf{L}$ and $\mathbf{U} - \mathbf{\Sigma}$ are positive semidefinite, respectively. (The unconstrained maximum likelihood estimator is proportional to the usual sample covariance matrix.) The method described here is based on a reparametrization of the Wishart distribution. Our numerical approach compares favorably to the iterative method proposed by Calvin & Dykstra (Calvin & Dykstra, 1992). We present R implementations of both optimization algorithms, and overview further extensions to two multivariate normal samples.

References

J. A. Calvin & R. L. Dykstra (1992). A note on maximizing a special concave function subject to simultaneous Loewner order constraints. *Linear Algebra and Its Applications*, 176, 37–42.