Electrical Load Forecasting in R

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Liberalisation of the Italian electricity market

- Introducing liberalisation of the Italian electricity sector
  (Italy: Legislative Decree 79/99)
- Creating conditions more conducive to fair competition and putting in place a true internal market
  (EU: Directive 2003/54/EC)

Opening up of the internal market allows customers to choose the most attractive electricity suppliers.

Liberalised Italian electricity market

- Power system: grid system
- Appropriate energy balance in the network
- Grid managers penalise the energy suppliers which do not contribute to the local grid with as much energy as needed by their own customers

→ Necessity of an effective **energy load model** to make medium term forecasts on an hourly basis

Six market zones

Source: GME (2004)

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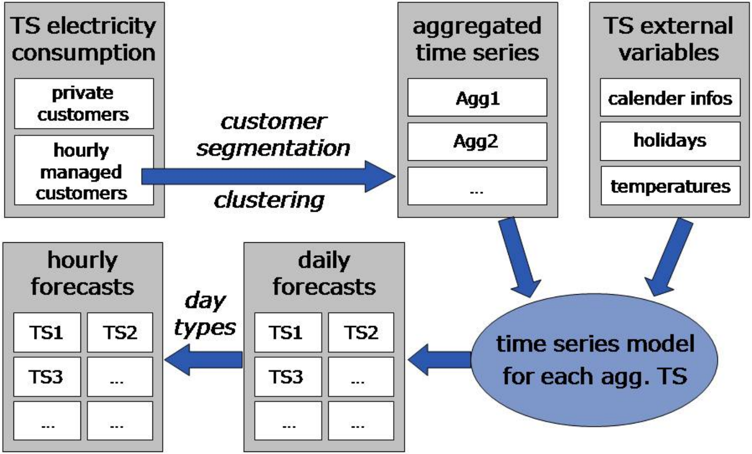
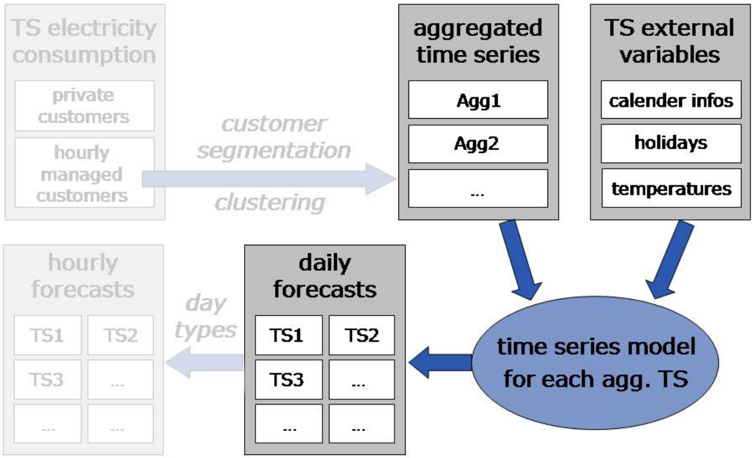
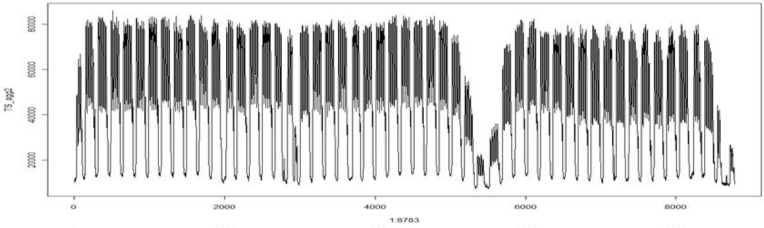
Forecasting procedure



Forecasting procedure



Exemplary electricity consumption (1 year)



Obvious influence of

- day of the week
- hour
- holidays

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Time series models

We have

An univariate time series of aggregated electricity consumption

$$Y_1, \ldots, Y_n$$

A multivariate time series of the external regressors

$$\mathbf{x}_1, \ldots, \mathbf{x}_n$$

with $\mathbf{x}_t = (x_{t,1}, \ldots, x_{t,k})'$ for $t = 1, \ldots, n$

We need

A model that connects the time series

$$Y_t = Y_t(x_{t,1}, \ldots, x_{t,k}, Y_{t-1}, Y_{t-2}, \ldots)$$

Time series models

1 General model

$$Y_t = \mathbf{x}_t' \beta + W_t$$

2 Autoregressive model with external regressors

$$Y_t = \mathbf{x}_t' \beta + \sum_{i=1}^{p} \phi_i Y_{t-i} + \epsilon_t, \quad \epsilon_1, \dots, \epsilon_n \text{ i.i.d. } N(0, \sigma^2)$$

3 Regression model with autoregressive error

$$Y_t = \mathbf{x}_t' \beta + W_t, \quad W_t = \sum_{i=1}^{p} \phi_i W_{t-i} + \epsilon_t, \quad \epsilon_1, \dots, \epsilon_n \text{ i.i.d. } N(0, \sigma^2)$$

1 General model

Model

$$Y_t = \underbrace{\beta_1 x_{t,1} + \ldots + \beta_k x_{t,k}}_{\text{external regressors}} + \underbrace{W_t}_{\text{error}}, \qquad W_t \text{ not specified}$$

Exponential smoothing with external regressors
(prediction-orientated, model is not estimated)
unknown error W_t is predicted with exponential smoothing:

$$\tilde{W}_{t+1} = \alpha W_t + (1-\alpha)\tilde{W}_t$$

Least squares approach, suggested by Wang (2006)

$$\sum_t (Y_t - \tilde{Y}_t)^2 \rightarrow \min_{\alpha,\boldsymbol{\beta}}!$$

Model

$$Y_t = \underbrace{\beta_1 x_{t,1} + \ldots + \beta_k x_{t,k}}_{\text{external regressors}} + \underbrace{\phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p}}_{\text{AR term}} + \underbrace{\epsilon_t}_{\text{error}}$$

- The R function *arima()* assumes this model.
- The order p of the AR process has to be known.
- The regression coefficients $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)'$ and the AR parameters $\boldsymbol{\phi} = (\phi_1, \ldots, \phi_p)'$ are estimated simultaneously by linear regression.

3 Regression model with autoregressive error

Model

$$Y_t = \mathbf{x}_t'\boldsymbol{\beta} + W_t, \quad W_t = \sum_{i=1}^{p} \phi_i W_{t-i} + \epsilon_t, \quad \epsilon_t \text{ i.i.d. } \mathrm{N}(0, \sigma^2)$$

$$\mathbf{Y} = X\boldsymbol{\beta} + \mathbf{W}, \quad \mathbf{W} \text{ is AR process}, \quad \mathbf{W} \sim \mathrm{N}(\mathbf{0}, \Gamma(\boldsymbol{\phi}, \sigma^2))$$

unknown:

- order p of the AR process
- AR parameters $\boldsymbol{\phi} = (\phi_1, \ldots, \phi_p)'$
- regression coefficients $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)'$
- variance of white noise σ^2

3 Regression model with autoregressive error

Parameter estimation of ϕ, β, σ^2 (p assumed to be known)
Extension of the iterative Cochrane-Orcutt procedure

Step 0:

- Set $\Gamma^{(0)} := I$

Step i:

- Calculate $\hat{\beta}^{(i)} = \hat{\beta}_{GLS}(\Gamma^{(i-1)})$
- Let $\hat{\phi}^{(i)}$ be the ML estimates of ϕ in fitting an AR model to $W_t^{(i)} = Y_t - \mathbf{x}_t' \hat{\beta}^{(i)}$
- Calculate the covariance matrix $\Gamma^{(i)} = \Gamma(\hat{\phi}^{(i)})$

Step i is repeated until no further change occurs in the parameter estimates.

Order selection

- Choosing the order p of the AR process
- Deciding which of the regression coefficients are significant
- AIC, AICC, BIC, FPE
- New development: FPE* (allows external regressors)

Forecasting process

	General model (exp. smoothing)	Regression model with AR error
Model	$\mathbf{Y} = \mathbf{X}'\boldsymbol{\beta} + \mathbf{W}$	$\mathbf{Y} = \mathbf{X}'\boldsymbol{\beta} + \mathbf{W}$, $\mathbf{W}$ is AR process
Unknown parameters	α, $\boldsymbol{\beta}$	p, $\boldsymbol{\phi}$, $\boldsymbol{\beta}$
Estimation method	Minimisation of First Step Prediction Error	Maximum likelihood
Order selection	External regressors	External regressors, AR length p

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Using R

How we use R

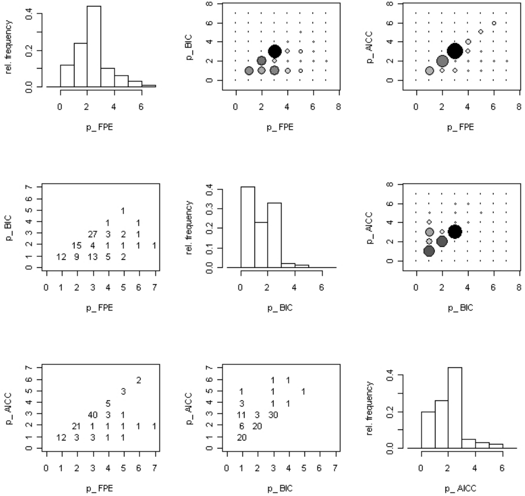
- Automated model selection
  - Implementation of the extended Cochrane-Orcutt algorithm
  - Comparison of order selection criteria
- Data simulations to test the automated model selection

Future steps

- Implementation of the Wang algorithm (exponential smoothing)
- Forecasting and prediction intervals

Exemplary graphical output

(comparison of order selection criteria)



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Thank you for your attention.