R-Packages for Robust Asymptotic Statistics

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joint work with
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useR! – The R User Conference 2008
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Outline

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Setup I

Ideal model: L_2-differentiable parametric family of probability measures, parameter space: $\Theta \subset \mathbb{R}^k$ (open)

Estimator class: asymptotically linear estimators (ALEs) S_n

$$S_n(x_1, \dots, x_n) = \theta + \frac{1}{n}\sum_{i=1}^{n} \psi_\theta(x_i) + R_n$$

$x_1, \dots, x_n$: sample

ψ_θ: influence curve/function (IC) at $\theta \in \Theta$

R_n: asymptotically negligible remainder

E.g. as. normal M-, L-, R-, S- and MD-estimators

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Setup II

Infinitesimal neighborhood: deviations (gross errors, outliers, etc.) from the ideal model P_θ of form

$$d_*(P_\theta, Q) = \frac{r}{\sqrt{n}} =: r_n \qquad Q \in \mathcal{M}_1$$

$\mathcal{M}_1$: set of all probability measures

d_*: some distance or pseudo-distance

r: radius in $[0, \sqrt{n}]$

E.g. Tukey's gross error model

$$Q = (1 - r_n)P_\theta + r_n H_n \qquad H_n \in \mathcal{M}_1$$

Optimally robust ALEs

Optimization problem:

$$\mathrm{G}\big(\mathrm{asBias}\,(\mathrm{S_n}), \mathrm{asVar}\,(\mathrm{S_n})\big) = \text{min!}$$

G: positive, convex, strictly increasing in both args

$\mathrm{asBias}\,(\mathrm{S_n})$: some function of ψ_θ (IC)

$\mathrm{asVar}\,(\mathrm{S_n})$: some function of ψ_θ (IC)

Hence: minimum is taken over all ICs ψ_θ

Optimal solutions: Rieder (1994) [3], Ruckdeschel and Rieder (2004) [10], Kohl (2005) [2]

Unknown radius: radius-minimax estimator; cf. Rieder et al. (2008) [8]

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Optimally robust estimation

Possible steps to compute an optimally robust estimator:

1. Decide on ideal model, neighborhood and risk
2. Try to find a rough estimate for the amount $r_n \in [0,1]$ of gross errors such that $r_n \in [\underline{r_n}, \overline{r_n}]$.
3. Choose and evaluate appropriate initial estimate; e.g., Kolmogorov or Cramér von Mises MD-estimator
4. Estimate the parameter(s) of interest by means of the corresponding radius-minimax estimator (cf. Rieder et al. (2008) [8]) using a k-step ($k \geq 1$) construction.

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Some examples

- Normal (Gaussian): location and scale
- Binomial: probability of success
- Poisson: positive mean
- Gamma: shape and scale
- Gumbel: location and scale
- all smoothly parameterized exponential families of full rank

- Approach also works for other smoothly parametrized families!

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Basic R-Packages

- distr: S4-classes for distributions.
- distrEx: Functionals on distributions.
- RandVar: S4-classes and methods for random variables.
- distrMod: S4-classes for parametric families of probability measures, minimum distance (MD) estimators.
- RobAStBase: S4-classes for ICs and infinitesimal neighborhoods.

cf. Ruckdeschel et al. (2006) [9], Kohl (2005) [2], `http://r-forge.r-project.org/projects/distr/`,

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R-Packages for optimally robust estimation

Devel version 0.6 (version 0.5 on CRAN)

ROptEst: Optimally robust estimation for L2 differentiable parametric families.

RobLox: Optimally robust estimation for normal (Gaussian) location and scale (optimized for speed).

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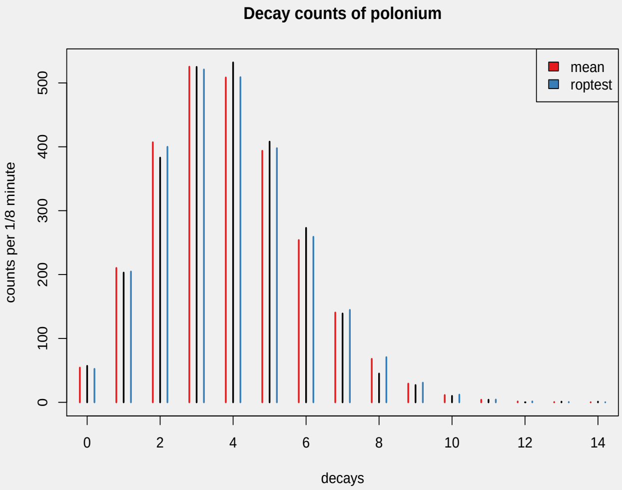
Example 1: Poisson

Decay counts of polonium by Rutherford and Geiger (1910); cf. Feller (1968)[1]

```
R > table(x)

x
  0   1   2   3   4   5   6   7   8   9  10  11  13  14
 57 203 383 525 532 408 273 139  45  27  10   4   1   1

R > ## ML-estimate
R > mean(x)

[1] 3.871549

R > ## or with package distrMod
R > MLest <- MLEstimator(x, PoisFamily(), interval = c(0, 10))
R > estimate(MLest)

  lambda
3.871547

R > ## Optimally robust 3-step estimate from package ROptEst (version 0.6.0)
R > ## takes about 4 sec (Centrino Duo 1.66 GHz)
R > ROest <- roptest(x, PoisFamily(), eps.upper = 0.05, interval = c(0, 10), steps = 3)
R > estimate(ROest)

  lambda
3.907973
```

Example 1: Poisson - comparison of results

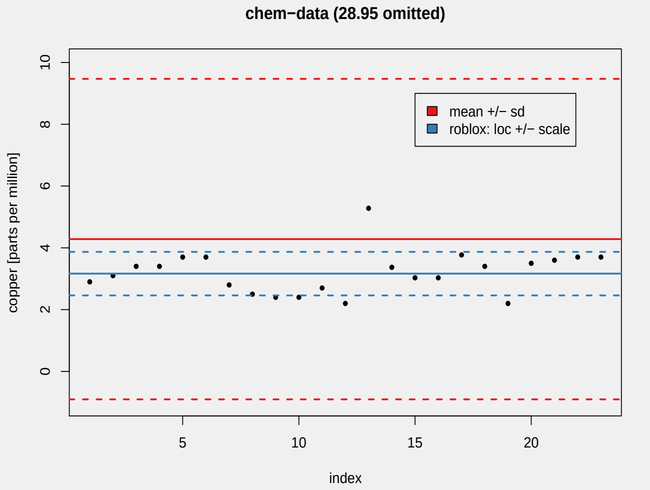


Example 2: Normal location and scale

Copper in wholemeal flour; cf. MASS [4]

```
R > chem

 [1]  2.90  3.10  3.40  3.40  3.70  3.70  2.80  2.50  2.40  2.40  2.70  2.20
[13]  5.28  3.37  3.03  3.03 28.95  3.77  3.40  2.20  3.50  3.60  3.70  3.70

R > ## ML-estimate (mean and sd) from package distrMod
R > MLest <- MLEstimator(chem, NormLocationScaleFamily())

R > ## median and MAD
R > initial.est <- c(median(chem), mad(chem))

R > ## Optimally robust 3-step estimate from package ROptEst (version 0.6.0)
R > ## takes about 80 sec (Centrino Duo 1.66 GHz)
R > ROest1 <- roptest(chem, NormLocationScaleFamily(), eps.upper = 0.05, steps = 3,
+                     initial.est = initial.est)

R > ## Use package RobLox (version 0.6.0) which is optimized for speed!
R > ## takes about 0.12 sec (Centrino Duo 1.66 GHz)
R > ROest2 <- roblox(chem, eps.upper = 0.05, k = 3, returnIC = TRUE)
```

Example 2: Normal location and scale



Example 3: Affymetrix gene expression data

Extract log-PM (perfect match) data from a HG U133+ 2.0 array

```
R > library(MAQCsubsetAFX)
R > data(refA)
R > ex.data <- refA[,1]
R > CDFINFO <- getCdfInfo(ex.data)
R > ids <- featureNames(ex.data)
R > INDEX <- sapply(ids, get, envir = CDFINFO)
R > NROW <- unlist(lapply(INDEX, nrow))
R > table(NROW)

NROW
    8     9    10    11    13    14    15    16    20    69
    5     1     6 54130     4     4     2   482    40     1

R > rawData <- intensity(ex.data)
R > fun <- function(INDEX, x) log2(x[INDEX[,1], ])
R > logPM <- lapply(INDEX, fun, x = rawData)
```

Example 3: Affymetrix gene expression data

Optimally robust estimation of location and scale for each Affymetrix ID via roblox and rowRoblox

```
R > ## takes about 17 minutes (Centrino Duo 1.66 GHz)
R > ROest1 <- lapply(logPM, function(x) estimate(roblox(x)))

R > ## takes about 1.3 sec (Centrino Duo 1.66 GHz)
R > nr <- as.integer(names(table(NROW)))
R > ROest2 <- matrix(NA, ncol = 2, nrow = length(NROW))
R > for(k in nr){
+     ind <- which(NROW == k)
+     temp <- do.call(rbind, logPM[ind])
+     ROest2[ind, 1:2] <- estimate(rowRoblox(temp))
+ }

R > ## maximum deviation roblox vs. rowRoblox: location
R > max(abs(unlist(ROest1)[seq(1, 2*54675-1, 2)] - ROest2[,1]))

[1] 5.640855e-06

R > ROest12 <- unlist(ROest1)[seq(2, 2*54675, 2)]

R > ## maximum deviation roblox vs. rowRoblox: scale
R > max(abs(unlist(ROest1)[seq(2, 2*54675, 2)] - ROest2[,2]))

[1] 2.591696e-06
```

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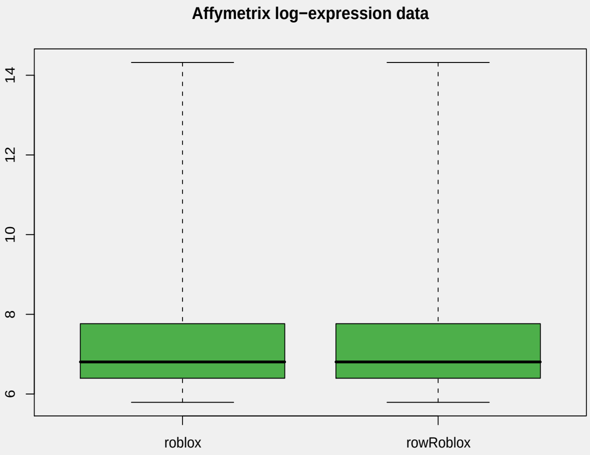

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Regression-Type Models

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ROptRegTS: Optimally robust estimation for regression and time series models.

RobRex: Optimally robust estimation for linear regression with normal errors.

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Current developments

- Confidence intervals
- Diagnostic plots
- Simpler user interfaces for regression models

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Bibliography I

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