Bayesian Covariance Selection in Hierarchical Linear Mixed Models

Regina Tüchler, Department of Statistics and Mathematics, WU Vienna

Sylvia Frühwirth-Schnatter, Department of Applied Statistics and Econometrics, JKU Linz

The Random-Effects Model

In the centered parameterization the random effects model writes:

$$y_i = Z_i\beta_i + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}_{T_i}(0, \sigma^2_\varepsilon I),$$
$$\beta_i = \beta^G + u_i, \quad u_i \sim \mathcal{N}_d(0, Q).$$

y_i . . . vector of T_i repeated measurements for subject i,
Z_i . . . design matrix ($T_i \times d$),
β_i . . . vector of d random effects with mean β^G and covariance matrix Q,
ε_i . . . model error with variance σ^2_ε

The Random-Effects Model

We apply the Cholesky decomposition to the variance-covariance matrix Q:

$$Q = CC', \quad C \quad \text{lower triangular}.$$

Therefore the random effects equal:

$$\beta_i = \beta^G + C\tilde{z}_i, \quad \tilde{z}_i \sim \mathcal{N}_d(0, I).$$

The model in the non-centered parameterization:

$$y_i = Z_i\beta^G + Z_iC\tilde{z}_i + \varepsilon_i, \quad \tilde{z}_i \sim \mathcal{N}_d(0, I), \quad \varepsilon_i \sim \mathcal{N}_{T_i}(0, \sigma^2_\varepsilon I).$$

Covariance Selection

The elements C of the covariance matrix appear as regression coefficients in the model equation. Therefore common variable selection tools may be applied to select elements in C.

We define indicators γ_{lm} for the $d \cdot (d+1)/2$ lower triangular elements of C.

$$C_{lm} = 0, \quad \text{iff } \gamma_{lm} = 0,$$
$$C_{lm} \neq 0, \quad \text{iff } \gamma_{lm} = 1,$$
$$\text{for } l \geq m.$$

If, for example all elements in the l-th row, $C_{l:}$, are 0, the l-th random effect β_{il} is shrunk toward a fixed effect.

The MCMC algorithm

Step I: $\gamma_{lm}|\gamma_{\setminus lm}, \beta^G, \tilde{z}, \sigma_\varepsilon^2, y$ discrete with 2 possible outcomes.

Step II: Only the non-zero elements $C^\gamma|\gamma, \beta^G, \tilde{z}, \sigma_\varepsilon^2, y$ from a multivariate normal distribution.

Step III: $\beta^G|C^\gamma, \sigma_\varepsilon^2, y$ from a multivariate normal distribution based on the the marginal heteroscedastic model.

Step IV: $\tilde{z}|\beta^G, C^\gamma, \sigma_\varepsilon^2, y$ from a multivariate normal distribution.

Step VI: $\sigma_\varepsilon^2|\beta^G, C^\gamma, \tilde{z}, y$ from an inverted Gamma distribution.

Summary

- The new method makes it possible to start with a very general model.
- We may determine fixed effects and select a sparse structure in the covariance matrix of the remaining random effects.
- Improved convergence behaviour because we avoid to estimate over-fitted models.