Graphical Exploratory Analysis Using Halfspace Depth

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Bivariate halfspace depth (Tukey depth)

Take a fixed collection of **datapoints**:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n).$$

Given an arbitrary point (x, y):

take all (closed) halfspaces having (x, y) on their boundary;
count how many datapoints lie inside them;
take the minimum of this count over the halfspaces.

That is: the bivariate halfspace depth of a point $\vartheta = (x, y)$ is the minimal number of the datapoints lying in a closed halfspace containing ϑ (on its boundary).

$$\boxed{D(\vartheta) = \inf_{u \neq 0} \#\{i\colon u^T(z_i - \vartheta) \geqslant 0\},}$$

where $z_i = (x_i, y_i)$, $\vartheta = (x, y)$, and $\#\{\cdot\} = \mathrm{card}\{\cdot\}$.

Depth = 0 (movie)

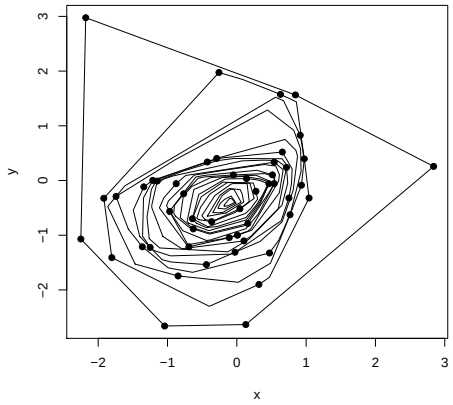
Depth = 1 (movie)

Depth = 2 (movie)

Tukey depth contours

Depth contour of level $k \equiv$ set of points with depth $\geqslant k$.

Nested, convex,...



Bagplot

Rousseeuw, Ruts, and Tukey (1999): a bivariate boxplot

Bag: depth contour containing about 1/2 of observations

Tukey median: a point selected from the contour with maximal depth (various methods possible, the Steiner point is our choice)

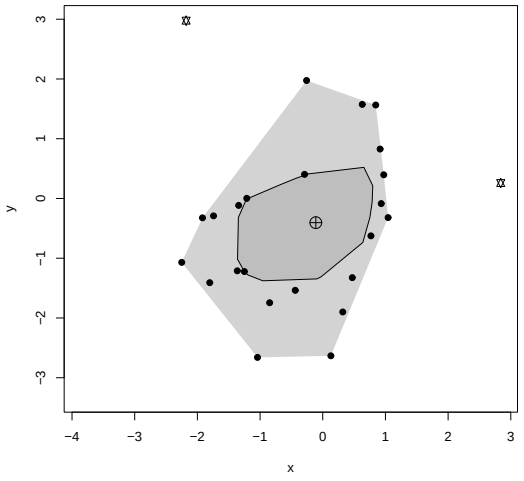
Fence: magnified bag (by fudge factor 3, with Tukey median as center)

Outliers: datapoints outside the fence

Loop: the convex hull of the datapoints inside the fence

Bagplot in action

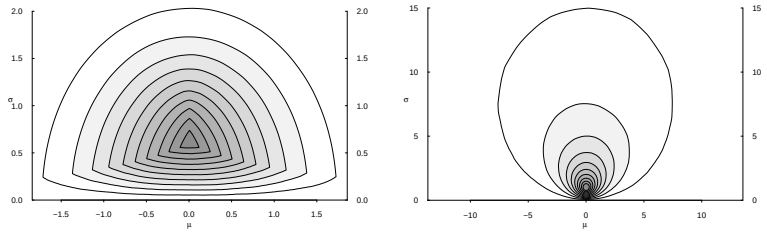
```
> library(depth)
> bagplot(x,y)
```



Student depth (location-scale)

Rousseeuw and Hubert (1998), Mizera (2002).

Mizera and Müller (2004): halfspace depth in the Lobachevski geometry of the location-scale space (a shortest, but perhaps not the most understandable definition).

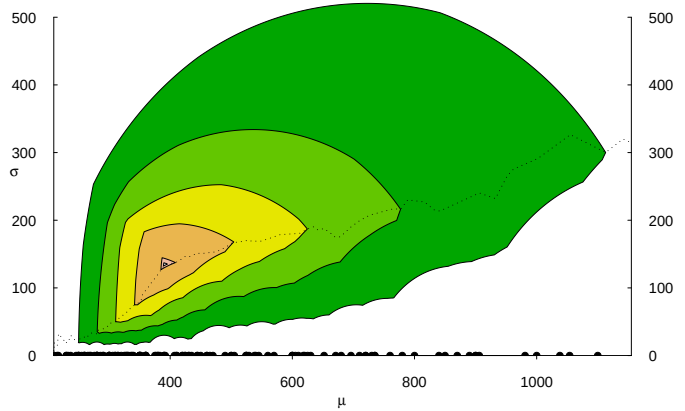



```
> plot(lsdc(rnorm(100000),'dozen'),maxline=F)
> plot(lsdc(rt(100000,1),'dozen'),maxline=F)
```


Depth = 2 (movie)

Student depth contours

```
> plot(lsdc(rivers,"six",maxline = T),paint=terrain.colors(6))
> points(rivers,rivers*0,pch=16)
```



Computer science

In general, NP hard. But plotting fortunately only dim 2.

Student depth contours: $\mathrm{O}(\mathfrak{n})$, apart from the initial $\mathrm{O}(\mathfrak{n} \log \mathfrak{n})$ sorting.

Tukey depth: all contours $\mathrm{O}(\mathfrak{n}^2)$ (but who needs them all?)

Individual depth contours: better? Yes - at least in theory...

Practical algorithm (jointly with David Eppstein): a dynamic convex hull structure (updating strategy).

Implementation: R / ... ?

Interpreted languages (Matlab, R, Python, Lisp) are fun ...

... but slow. Compiled languages (machine code, assembly, FORTRAN, C(++), Java) are fast...

... but are work (= no fun).

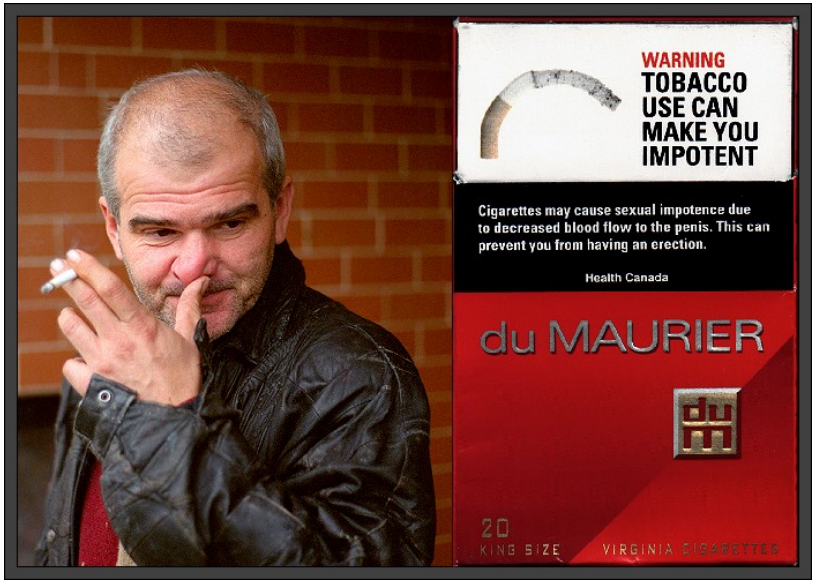
A case study of useR psychoanalysis ($n = 1$)

- FORTRAN avoided (trauma from childhood).
- C routines running (translated from MATLAB, a labor therapy).
- Python prototypes of my co-author David Eppstein deciphered (still waking up at night).
- `Segmentation fault` for $n > 100000$ taken care of (thanks to Duncan Temple Lang for the `S_alloc` command!)
- The next use of `S_alloc` command successfully guessed (without finding any documentation or asking DTL once again).
- Poor Man's Zoom - a Wittgensteinian approach to graphics.
- Eventually, learned how to pass R CMD check (man gets accustomed even to gallows, a Slovak proverb).
- And never ever asked anything on `R-help`.
- It's almost done. (By the anniversary of October revolution?)

Frustrations of a random sample unit: in the search of identity

- (Pressburger blut or Midwesterner in a broad sense?.)
- Computational statistician? Oh, no FORTRAN, thanks...
- UseR from 1998? Bring two witnesses, please.
(UseR $<$ 2000 $\approx$ NSDAP $<$ 1933 or
Czechoslovak Communist Party $<$ 1948)
- Besides, useRs don't worry about things like segmentation faults and `S_alloc` documentation.
- DevelopeR then? Oh, don't make me blushing...
- **AbuseR**. Self-promotion, albeit with attacks of guilty feelings (will a confession get me a pardon?).
- "Don't work on software, work on ideas" (Rich Sutton, a computer science Zen Master from Edmonton).

Warning



Warning

ALTHOUGH ABUSING R
WAS NOT PROVED TO BE ADDICTIVE,
IT SHOULD BE NOTED
THAT IT OFTEN LEADS TO HARDER STUFF.

Viennese epilogue

Stefan Zweig

Theodor Herzl

Some ideas carry a lot of power...

...and the genie is out of the bottle.

Also:

"That what is, often prevails over what could, or even over what should be."

Is it Fellini? (A reward offered for help with this.)