Flexible, optimal matching for observational studies

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Outline

Optimal matching of two groups

- Comparing nuclear plants: an illustration

Generalizations of pair matching

The R implementation

Existing site

	date	capacity
A	2.3	660
B	3.0	660
C	3.4	420
D	3.4	130
E	3.9	650
F	5.9	430
G	5.1	420

New site

	date	capacity
H	3.6	290
I	2.3	660
J	3.0	660
K	2.9	110
L	3.2	420
M	3.4	60
N	3.3	390
O	3.6	160
P	3.8	390
Q	3.4	130
R	3.9	650
S	3.9	450
T	3.4	380
U	4.5	440
V	4.2	690
W	3.8	510
X	4.7	390
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"date" is date of construction, in years after 1965; "capacity" is net capacity of the power plant, in MWe above 400.

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New and refurbished nuclear plants: discrepancies in capacity and year of construction

Existing	New sites																		
	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	28	0	3	22	14	30	17	28	26	28	20	22	23	26	21	18	34	40	28
B	24	3	0	22	10	27	14	26	24	24	16	19	20	23	18	16	31	37	25
C	10	18	14	18	4	12	6	11	9	10	14	12	6	14	22	10	16	22	28
D	7	28	24	8	14	2	10	6	12	0	24	22	4	24	32	20	18	16	38
E	17	20	16	32	18	26	20	18	12	24	0	2	20	6	8	4	14	20	14
F	20	31	28	35	20	29	22	20	14	26	12	9	22	5	15	12	9	11	12
G	14	32	29	30	18	24	17	16	10	22	12	10	17	6	16	14	4	8	17

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Optimal vs. Greedy matching

By evaluating potential matches all together rather than sequentially, optimal matching (blue lines) reduces the sum of distances from 82 to 63.

Introducing restrictions on who can be matched to whom: calipers

In the nuclear plants example, suppose we choose to insist upon a caliper of three years in the date of construction. This would forbid five potential matches, indicated below in red.

Exist-ing	New sites																		
	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	28	0	3	22	14	30	17	28	26	28	20	22	23	26	21	18	34	40	28
B	24	3	0	22	10	27	14	26	24	24	16	19	20	23	18	16	31	37	25
C	10	18	14	18	4	12	6	11	9	10	14	12	6	14	22	10	16	22	28
D	7	28	24	8	14	2	10	6	12	0	24	22	4	24	32	20	18	16	38
E	17	20	16	32	18	26	20	18	12	24	0	2	20	6	8	4	14	20	14
F	20	31	28	35	20	29	22	20	14	26	12	9	22	5	15	12	9	11	12
G	14	32	29	30	18	24	17	16	10	22	12	10	17	6	16	14	4	8	17

Introducing restrictions on who can be matched to whom: calipers

With `optmatch`, matches are forbidden by placing ∞'s in the distance matrix.

Exist-ing	New sites																		
	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	28	0	3	22	14	30	17	28	26	28	20	22	23	26	21	18	34	Inf	Inf
B	24	3	0	22	10	27	14	26	24	24	16	19	20	23	18	16	31	37	Inf
C	10	18	14	18	4	12	6	11	9	10	14	12	6	14	22	10	16	22	28
D	7	28	24	8	14	2	10	6	12	0	24	22	4	24	32	20	18	16	38
E	17	20	16	32	18	26	20	18	12	24	0	2	20	6	8	4	14	20	14
F	20	Inf	28	Inf	20	29	22	20	14	26	12	9	22	5	15	12	9	11	12
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Example # 2: Gender equity study for research scientists[1]

Women and men scientists are to be matched on grant funding.

Women		Men	
Subject	$\log_{10}$(Grant)	Subject	$\log_{10}$(Grant)
A	5.7	V	5.5
B	4.0	W	5.3
C	3.4	X	4.9
D	3.1	Y	4.9
		Z	3.9

[1] Discussed in Hansen and Klopfer (2005), Hansen (2004)

Full Matching[2] the Gender Equity Sample

Women		Men	
Subject	$\log_{10}$(Grant)	Subject	$\log_{10}$(Grant)
A	5.7	V	5.5
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		Z	3.9

- Similar to matching with replacement, but creates disjoint matched sets — better for tests & CIs.
- In contrast to pair matching, it finds matches for everyone with a suitable counterpart.
- In contrast to multiple controls matching, it doesn't force poor matches.
- In `optmatch`, can be combined with structural restrictions.

[2](Rosenbaum, 1991; Hansen and Klopfer, 2005)

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Subject	$\log_{10}$(Grant)	Subject	$\log_{10}$(Grant)
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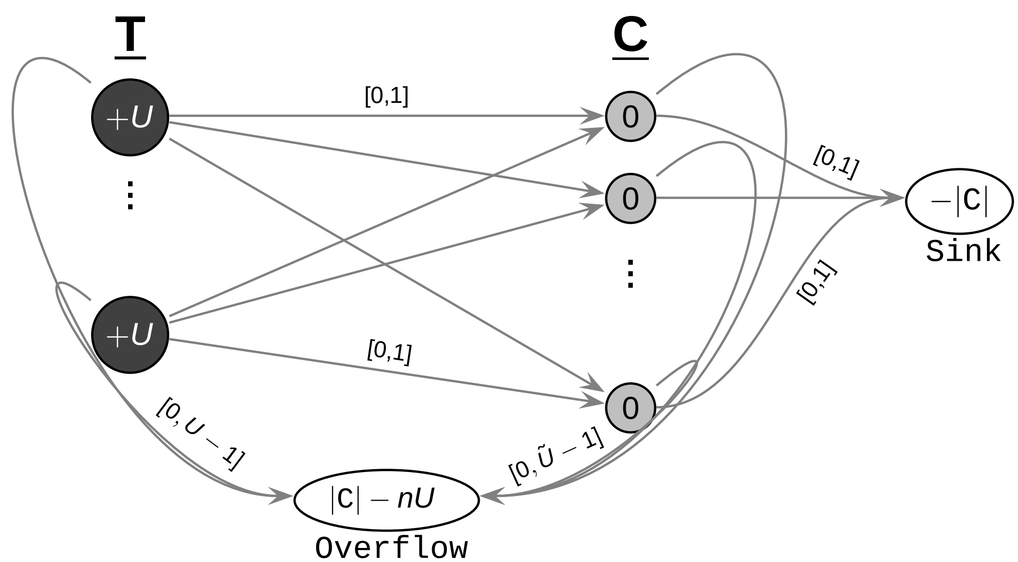
Comparing nuclear plants: an illustration

Generalizations of pair matching

The R implementation

Under the hood

Full matching via network flows[3]



[3](Hansen and Klopfer, 2005, Fig. 2). Time complexity of the algorithm is $O(n^3 \log(n \max(\mathrm{dist})))$.

Arguments to `fullmatch()`

distance The argument demanding most attention from the user, b/c it defines "good" matches and because very large distance matrices can tax R's memory limits. A new helper function, `makedist()`, eases both of these efforts.

min.controls, max.controls In propensity matching, can be important for efficiency — see Hansen (2004), Augursky and Kluve (2004).

omit.fraction for use in matched sampling (as opposed to matching all or most of a sample). Not needed for getting rid of subjects without suitable potential matches. If you're not specifically out to reduce the size of the control group, can be ignored.

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Summary

- With `optmatch`, R offers the most comprehensive optimal matching implementation for statistics.
- `fullmatch()` solves optimally such traditional problems as matched sampling, pair matching, and matching with *k* controls.
- `fullmatch()` can also solve matching problems flexibly, and far more effectively, by way of full matching, with or without structural restrictions (Hansen and Klopfer, 2005; Hansen, 2004).
- The effort required to code optimal and full matching algorithms seems to have dissuaded their widespread use. Now that I've made that effort, perhaps this situation can change! :)

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Example with propensity scores and stratification prior to matching

```
>nuclear$pscore <- glm(pr~.-cost,
+ family=binomial(),data=nuclear)$linear.predictors

> pscorediffs <- function(trtvar,data) {
+ pscr <- data[names(trtvar), 'pscore']
+ abs(outer(pscr[trtvar],pscr[!trtvar], '-'))
+ }

> psd2 <- makedist(pr~pt, nuclear, pscorediffs)

> fullmatch(psd2)

> fullmatch(psd2, min.controls=1, max.controls=3)
> fullmatch(psd2, min=1, max=c('0'=3, '1'=2))
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Jake Bowers' and my `RItools` package provides diagnostics. . .

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Modes of estimation for treatment effects

Preferred mode of inference	Type of outcome	
	Categorical	Continuous
Randomization	Agresti (2002), <u>Categorical Data Analysis</u>; Rosenbaum (2002a), "Atributing effects to treatment . . . "	Rosenbaum (2002c), <u>Observational Studies</u>; Rosenbaum (2002b), "Covariance adjustment . . . ."
Conditional [a]	Agresti (2002); Cox and Snell (1989), <u>Analysis of binary data</u>	ordinary OLS[b] is fine; see also Rubin (1979), "Using multivariate matched. . . ."
Bayes, esp. hierarchical linear models [c]	Agresti (2002)	Smith (1997), "Matching with multiple controls. . . "; Raudenbush and Bryk (2002), <u>Hierarchical linear models</u>

[a]Uses a fixed effect for each matched set.

[b]i.e., OLS with a fixed effect for each matched set plus treatment effect(s)

[c]Uses a random effect for each matched set.

Agresti, A. (2002), *Categorical data analysis*, John Wiley & Sons.

Augursky, B. and Kluve, J. (2004), "Assessing the performance of matching algorithms when selection into treatment is strong," Tech. rep., RWI-Essen.

Cox, D. R. and Snell, E. J. (1989), *Analysis of binary data*, Chapman & Hall Ltd.

Hansen, B. B. (2004), "Full matching in an observational study of coaching for the SAT," *Journal of the American Statistical Association*, 99, 609–618.

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Hansen, B. B. and Klopfer, S. O. (2005), "Optimal full matching and related designs via network flows," Tech. Rep. 416, Statistics Department, University of Michigan.

Raudenbush, S. W. and Bryk, A. S. (2002), *Hierarchical Linear Models: Applications and Data Analysis Methods*, Sage Publications Inc.

Rosenbaum, P. R. (1991), "A Characterization of Optimal Designs for Observational Studies," *Journal of the Royal Statistical Society*, 53, 597– 610.

— (2002a), "Attributing effects to treatment in matched observational studies," *Journal of the American Statistical Association*, 97, 183–192.

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Rubin, D. B. (1979), "Using Multivariate Matched Sampling and Regression Adjustment to Control Bias in Observational Studies," *Journal of the American Statistical Association*, 74, 318–328.

Smith, H. (1997), "Matching with Multiple Controls to Estimate Treatment Effects in Observational Studies," *Sociological Methodology*, 27, 325–353.